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Finite horizons, life-cycle savings, and time-series evidence on consumption

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This paper develops an explicitly aggregated life-cycle model which allows for finite horizons and declining individual labor supply. The model can generate a common upward trend in aggregate consumption, labor income, and nonhuman wealth, without relinquishing Hall's random walk at the micro level. Life-cycle factors are shown to imply that consumption changes should be (a) predictable and (b) smoother than in the infinite-horizon model. Econometric evidence using postwar U.S. data suggests that neither consumption's predictability nor its excess smoothness can be fully accounted for by life-cycle considerations.

1. Introduction

Since Hall (1978) much research in the consumption front has attempted to interpret the time series for *aggregate* consumption and savings as the solution to a dynamic optimization program solved by an *infinite-lived representative consumer* facing an exogenous, stochastic labor-income process.[1]

As is well known, tests of standard versions of the infinite-horizon model applied to postwar U.S. data reject some of its central predictions. Two findings are usually brought up as evidence against that model: (a) consump-

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[1]Such a model is also known in the literature as the Permanent Income model, a concept originally associated with Friedman (1957). Friedman's formulation is, however, much more general than the model used in the recent literature and in no case does it imply that aggregate time series be interpreted as the solution to the infinite-lived representative consumer program.

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tion is not a martingale [Flavin (1981)], and (b) consumption is *too smooth* relative to permanent income [Deaton (1987)].[2]

The present paper develops a framework for the time-series analysis of aggregate consumption which dispenses with the assumption of an infinite-lived representative consumer. The model shares the essential features of explicitly aggregated life-cycle models – as developed by Franco Modigliani and coauthors[3] – while preserving the convenience and tractability of the infinite-horizon model in terms of its econometric implementation.[4]

Life-cycle models of consumption typically stress the role of two factors which, by their very nature, are ignored in infinite-horizon models: (a) finite horizons and (b) a life-cycle profile for individual labor income characterized by retirement – or low labor income – in a late stage of the cycle. The latter feature is henceforth referred to as 'life-cycle savings', since it tends to induce additional savings in early stages of the life cycle. In the model below each of the two factors is regulated by a parameter. The infinite-horizon model appears as a special case, corresponding to a specific configuration of values for those parameters.

An important feature of the model is the existence of a *one-to-one mapping between the presence of finite horizons and/or life-cycle savings and the low frequency properties of aggregate labor income, consumption, and nonhuman wealth*. In particular, the model is capable of generating a common upward trend in those three aggregate series, without relinquishing Hall's martingale model at the micro level.

The paper also reexamines the evidence on 'predictability' and 'excess smoothness' of consumption in the context of the life-cycle model. Life-cycle considerations are shown to imply that consumption changes should be (a) predictable and (b) smoother than is implied by the infinite-horizon model. It is thus important to redesign the existing tests in order to filter out the influence of life-cycle factors.

The plan of the paper is as follows. Section 2 develops the basic life-cycle model. Section 3 examines its implications regarding the long-run behavior of aggregate time series. Section 4 derives the model's predictions on the predictability and variability of consumption changes. Econometric tests of

[2]The 'excess-smoothness' result arises when labor income is modeled as a unit-root process. When the same variable is modeled as being stationary around a deterministic trend, the opposite result obtains: consumption is too variable relative to permanent income. See Deaton (1987) for details.

[3]Modigliani (1986) offers an excellent review as well as complete references on the early life-cycle literature.

[4]Clarida (1988), in independent research, uses a traditional Modigliani life-cycle model with certain lifetimes and retirement period, in an attempt to explain the 'excess-smoothness' puzzle. Despite its usefulness in showing the potential role of life-cycle factors in that puzzle, as well in addressing other issues, the aggregation properties of the Modigliani life-cycle model make its econometric implementation rather burdensome except for some particular cases.

those predictions using postwar U.S. data are carried out in the same section. Section 5 summarizes and concludes.

2. A life-cycle model of aggregate consumption and savings

The model developed below is a discrete-time, quadratic-utility, open-economy version of the overlapping-generations framework in Blanchard (1985). Next I describe its basic ingredients.

2.1. Technology and international capital mobility

Domestic output Y is given by an aggregate production function $Y_t = K_t^{\alpha}(A_t L_t)^{(1-\alpha)}$, where K is aggregate capital, L denotes aggregate 'labor services' employed, and A is an exogenous process describing the technology level. We assume an open economy, with capital being perfectly (and instantaneously) mobile across economies. The world interest rate is constant and equal to r. Under the assumption of zero depreciation, and letting yl be the compensation per unit of labor services, profit maximization by competitive firms implies $r = \alpha(Y_t/K_t)$ and $yl_t = (1-\alpha)(Y_t/L_t)$. Using the production function above, and assuming a constant, exogenously given, value for L (which we normalize to be one), we obtain

$$yl_t = (1-\alpha)(\alpha/r)^{\alpha/(1-\alpha)} A_t,$$

i.e., aggregate labor income is proportional to the technology process A, and is thus determined independently of consumption/savings decisions.

2.2. Demographics and annuity markets

Regardless of his age, each consumer alive in period t faces a constant probability p of dying in period $t+1$, with $0 \leq p \leq 1$. As of t, the probability of being alive at $t+j$ is thus $(1-p)^j$.

The size of each cohort at birth is normalized to p, and is assumed to decline deterministically over time, at a rate also given by p. Using $N_{s,t}$ to denote the size, as of period t, of the cohort born in period s (where $t \geq s$), we have $N_{s,t} = p(1-p)^{(t-s)}$. Thus, total population at time t, $N_t \equiv \sum_{s=-\infty}^{t} N_{s,t}$, is constant and equal to 1.

Following Blanchard (1985) and Yaari (1965), we assume the existence of annuity markets whenever there is uncertainty about death (i.e., when $0 < p < 1$). Annuity firms make (receive) every period an annuity payment to (from) each consumer holding positive (negative) financial wealth, and inherit the wealth of that consumer at his death. A zero-profit condition in those markets, together with the population structure described above, implies an effective gross return on individual nonhuman wealth of $(1+z)$, where $(1+z) \equiv (1+r)(1-p)^{-1}$, with $(1+r)$ being the 'pure interest rate' and $(1-p)^{-1}$ the 'annuity rate'.[5]

When $p=0$, the consumer's horizon becomes infinite, no new cohorts are born (or, equivalently, they have zero size), and $z=r$ (since annuities do not exist). In that case the structure of the life-cycle economy collapses to the infinite-lived representative consumer economy. When $p=1$, each cohort lives for one period and is fully replaced by the next cohort in the following period. In the latter case the model is reduced to a sequence of 'static economies' with no savings.

2.3. Optimal individual behavior

In period t, each consumer born at time s maximizes his expected present discounted value of utility. Formally he solves:

$$\max \mathrm{E}_t \sum_{j=0}^{\infty} (1+\delta)^{-j}(1-p)^j U(c_{s,t+j}),$$

subject to

$$W_{s,t+1+j} = W_{s,t+j}(1+z) + yl_{s,t+j} - c_{s,t+j}, \tag{1}$$

$$\lim_{j\to\infty} (1+z)^{-j} W_{s,t+j} = 0, \tag{2}$$

for $j=0,1,2,\ldots$, and where c is consumption, W is nonhuman wealth, and yl is labor income. $x_{s,t}$ denotes the value of variable x at time t, for a consumer born in period s. δ is the discount rate. $\mathrm{E}_t x_{s,t+j}$ denotes the expected value of $x_{s,t+j}$ *conditional on the consumer being alive in period* $t+j$, given the information available at time t. (1) and (2) are the budget constraint and transversality condition, respectively. Individuals are assumed to be born with zero financial wealth, i.e., $W_{s,s}=0$.

Assuming quadratic utility and $r=\delta$, the first-order condition of the dynamic program above implies

$$\mathrm{E}_{t-1}\,\Delta c_{s,t} = 0, \tag{3}$$

i.e., *individual* consumption is a martingale with no drift.

[5]For a consumer dying in period t, death occurs before labor income is earned and goods consumed in that period, but after interest is earned on wealth invested at the beginning of the period. The presence of annuities (together with a zero-profit condition on annuity firms) can be seen as an effective transfer of wealth within members of the same cohort. Specifically, $W_{s,t-1}(1+r)pN_{s,t-1}$ is effectively transferred to the $(1-p)N_{s,t-1}$ members who remain alive, each thus getting $W_{s,t-1}(1+r)p/(1-p)$. Added to the usual return $W_{s,t-1}(1+r)$, this yields an effective return equal to $(1+r)(1-p)^{-1}$.

Applying (3) to the intertemporal budget constraint of our consumer – derived using (1) and (2) – we get the following expression for current individual consumption:

$$c_{s,t} = z[W_{s,t} + H_{s,t}], \tag{4}$$

where

$$H_{s,t} \equiv (1+z)^{-1} \sum_{j=0}^{\infty} (1+z)^{-j} \mathrm{E}_t yl_{s,t+j}$$

is a measure of human wealth in period t for a consumer born in period s. Notice that the model for the individual consumer here is formally identical to the standard representative consumer model. The only difference lies in the presence of an 'augmented' discount rate, which reflects the shorter expected horizons.

2.4. *Labor supply and labor income over the life cycle*

The amount of (effective) labor services supplied by an individual consumer, denoted by $L_{s,t}$, is assumed to decline geometrically over his lifetime at a rate α, reflecting underlying changes in his productivity and/or hours supplied. Specifically, we assume

$$L_{s,t} = (\Gamma/p)(1-\alpha)^{(t-s)},$$

where $0 \leq \alpha < 1$, and $\Gamma \equiv [1-(1-\alpha)(1-p)]$ satisfies the obvious aggregation/normalization restriction $1 \equiv L_t \equiv \sum_{s=-\infty}^{t} L_{s,t} N_{s,t}$, where we drop the s subscript to denote an aggregate variable. Obviously, the concept of declining labor supply is meaningless in the context of the representative consumer model. Thus, when $p = 0$ we will automatically assume $\alpha = 0$.

From the previous assumption it follows that[6]

$$yl_{s,t} = L_{s,t} yl_t = (\Gamma/p)(1-\alpha)^{(t-s)} yl_t . \tag{5}$$

[6]Given p, intergenerational distribution of aggregate labor income implied by (5) depends on a single parameter α. That specification is particularly convenient in the subsequent econometric implementation, but the nice aggregation properties of the Blanchard–Yaari framework do not depend on it. Given a general time-invariant labor-supply equation, $L_{s,t} = Q(s,t)$, it is not difficult to show that aggregation will go through as long as $Q(s,t)$ is a function of $(t-s)$ only – i.e., $Q = Q(t-s)$ – and satisfies $1 = \sum_{s=-\infty}^{t} Q(t-s) N_{s,t}$. In that case we can always express *aggregate* human wealth as $H_t = (1+z)^{-1} \sum_{j=0}^{\infty} (1+z)^{-j} M_j \mathrm{E}_t yl_{t+j}$, where $M_j \equiv p \sum_{s=-\infty}^{t} Q(t+j-s)(1-p)^{t-s}$. A class of functions satisfying the previous requirement is given by $Q(t-s) \equiv (1/p) \sum_{i=1}^{q} \Gamma_i (1-\alpha_i)^{t-s}$, where $\Gamma_i \equiv [1-(1-p)(1-\alpha_i)]/q$, $i = 1, \ldots, q$. As can be readily seen, the specification used in (5) corresponds to $q = 1$

2.5. *Aggregate behavior*

Expressions for the main aggregates can now be easily derived. Aggregate nonhuman wealth, $W_t \equiv \sum_{s=-\infty}^{t} W_{s,t} N_{s,t}$, will satisfy

$$W_t = W_{t-1}(1+r) + yl_{t-1} - c_{t-1}, \tag{6}$$

which captures the fact that, in the aggregate, the return on financial wealth is r instead of z, since annuity payments represent pure transfers among consumers.

Aggregate human wealth, $H_t \equiv \sum_{s=-\infty}^{t} H_{s,t} N_{s,t}$, will be given by

$$H_t = (1+z)^{-1} \sum_{j=0}^{\infty} (1+z)^{-j}(1-\alpha)^j \mathrm{E}_t yl_{t+j}, \tag{7}$$

where the equality follows (after some algebraic manipulation) from the definition of $H_{s,t}$ and (5).

Aggregate consumption, $c_t \equiv \sum_{s=-\infty}^{t} c_{s,t} N_{s,t}$, is given by

$$c_t = z(W_t + H_t). \tag{8}$$

Assuming that $\mathrm{E}(\Delta yl)$ exists and is equal to μ, we can rewrite (8) in the following convenient way:

$$c_t = \Omega + zW_t + \beta yl_t + u_t, \tag{9}$$

where

$$\beta \equiv \frac{z}{(z+\alpha)}, \qquad \Omega \equiv \frac{\beta\mu(1-\alpha)}{(z+\alpha)},$$

and

$$u_t \equiv \beta \sum_{j=1}^{\infty} (1+z)^{-j}(1-\alpha)^j(\mathrm{E}_t \Delta yl_{t+j} - \mu).$$

Notice that (9) establishes a linear relationship between aggregate consumption, labor income, and nonhuman wealth identical to the one postulated in the early life-cycle literature [e.g., Ando and Modigliani (1963)]. The nice aggregation properties of the Blanchard–Yaari framework generate a very simple relationship between the coefficients of the consumption equation and the underlying structural parameters. To the extent that aggregate labor income shows positive average growth ($\mu > 0$) the constant term Ω will be positive. In the presence of life-cycle savings ($\alpha > 0$), β – the 'marginal propensity to consume' out of yl in (9) – will be less than one. Note also that the disturbance term u_t has, by construction, zero mean, but will typically be serially correlated and correlated with W_t and yl_t. By setting $p = \alpha = 0$ the model yields the infinite-horizon consumption function $c_t = (\mu/r) + rW_t + yl_t + u_t$.[7]

Given initial values for yl and W, the joint behavior of aggregate consumption, non-human wealth and labor income is fully characterized by (6), (9), and a stochastic process for aggregate labor income.

3. Finite horizons, life-cycle savings, and long-run behavior of macroeconomic time series

This section examines the implications of the life-cycle model developed above on the long-run behavior of aggregate consumption, labor income, nonhuman wealth, and savings. Much of the discussion is carried out under the maintained hypothesis that aggregate labor income is a unit-root process with (positive) drift, an assumption to be justified below on empirical grounds.

3.1. *Long-run behavior of consumption*

As noted above, the assumption of equality between the discount rate and the return on financial wealth implies that changes in *individual* consumption are a martingale difference process with zero mean, independently of the stochastic properties of labor income. That result does *not* carry over to *aggregate* consumption. As is shown next, and as long as $p > 0$, aggregate consumption will show a trend closely related to that in aggregate labor income.

To see this, notice that aggregate consumption in period t can be decomposed as follows:

$$c_t = \sum_{s=-\infty}^{t-1} c_{s,t} N_{s,t} + N_{t,t} c_{t,t}.$$

Applying the E_{t-1} operator to both sides of the previous expression and using the martingale result above, we get:

$$\mathrm{E}_{t-1} c_t = (1-p) c_{t-1} + p \mathrm{E}_{t-1} c_{t,t}.$$

Thus $\mathrm{E}_{t-1} c_t$ has two components. The first one corresponds to expected one-period-ahead consumption by those alive at $t-1$ who will remain alive

[7]When $p = 1$, z is not well defined and, as a result, neither is eq. (9) In that case, the aggregate consumption function follows trivially from the consumer's budget constraint, and is given by $c_t = yl_t$.

at t. The second component reflects the level of consumption by the new cohort born at t, as expected at $t-1$. The assumption that individuals are born with zero nonhuman wealth implies that $c_{t,t} = zH_{t,t}$. Using the expression for individual human wealth above we can derive, after some manipulation,

$$c_t = (1-p)c_{t-1} + \Gamma\beta yl_{t-1} + \Gamma(1+z)(1-\alpha)^{-1}[\Omega + u_{t-1}] + \eta_t, \tag{10}$$

where $\eta_t \equiv c_t - \mathrm{E}_{t-1}c_t$ is the innovation in aggregate consumption, and Γ, Ω, β, and u are defined as above. Two parameter configurations are relevant:

Case I: $p = \alpha = 0$ (infinite horizons). In this case $\Gamma = 0$ and (10) collapses to Hall's (1978) martingale result $\Delta c_t = \eta_t$. The trends in consumption and labor income are 'decoupled': aggregate consumption follows a random walk, independently of aggregate labor income's low frequency properties.

Case II: $p > 0$, $\alpha \geq 0$ (finite horizons, with or without life-cycle savings). In this case $\Gamma > 0$, and we can express c_t as an infinite distributed lag of past yl's plus a zero mean disturbance term:

$$c_t = \Gamma\Omega(1+z)/p(1-\alpha) + \Gamma\beta \sum_{j=0}^{\infty} (1-p)^j yl_{t-1-j} + \nu_t, \tag{11}$$

where

$$\nu_t \equiv \sum_{j=0}^{\infty} (1-p)^j \left[(\Gamma(1+z)/(1-\alpha))u_{t-1-j} + \eta_{t-j}\right].$$

It is clear from (11) that, as long as $p > 0$, aggregate consumption's trend will share the features of aggregate labor income's trend. Thus, if yl is an $I(1)$ process with drift, then c will also be. First-differencing both sides of (11) and applying the unconditional expectation operator yields:

$$\mathrm{E}(\Delta c) = (\Gamma\beta/p)\mu,$$

i.e., finite horizons imply that average changes in c are proportional to average changes in yl. In the absence of life-cycle savings ($\alpha = 0$), we have $\beta = 1$ and $\Gamma = p$, thus implying $\mathrm{E}(\Delta c) = \mu$.[8]

[8]For small values of α, $\mathrm{E}(\Delta c)$ will be very close to μ.

The nature of the link between yl and c when $p > 0$ is now made more precise. By subtracting c_{t-1} from both sides of eq. (10):

$$\Delta c_t = -pc_{t-1} + \Gamma\beta yl_{t-1} + (\Gamma(1+z)/(1-\alpha))[\Omega + u_{t-1}] + \eta_t. \tag{12}$$

If yl is an $I(1)$ process, both Δc and u are $I(0)$, so it must be the case that $(\Gamma\beta yl_{t-1} - pc_{t-1})$ is also $I(0)$. In other words, c and yl must be cointegrated – in the sense of Engle and Granger (1987) – with a cointegrating vector $[1, -\Gamma\beta/p]$. In the particular case of $\alpha = 0$, that cointegrating vector simplifies to $[1, -1]$.

3.2. Long-run behavior of aggregate wealth and savings

Using (6) and (9) the following expression relating aggregate nonhuman wealth to aggregate labor income can be derived:

$$W_{t+1} = [1 - (z - r)]W_t - \Omega + (1 - \beta)yl_t - u_t. \tag{13}$$

It is convenient to consider the following three cases:

Case I: $p = \alpha = 0$ (infinite horizons). In this case $r = z$, $\beta = 1$, and $\Omega = \mu/r$. Accordingly, (13) simplifies to Campbell (1987)'s 'rainy day' equation:[9]

$$s_t \equiv \Delta W_{t+1} = -\mu/r - u_t,$$

i.e., savings are stationary and their unconditional expectation is proportional to average labor income growth, *though with the opposite sign*. Accordingly, when yl is $I(1)$ with positive drift, nonhuman wealth is $I(1)$ with negative drift.

Case II: $p > 0$, $\alpha = 0$ (finite horizons, no life-cycle savings). In this case $z > r$, $\beta = 1$, $\Omega = \mu/z$, and (13) can be rewritten as:

$$W_t = -\frac{\mu}{z(z-r)} - \sum_{j=0}^{\infty} [(1-(z-r)]^j u_{t-1-j}.$$

[9]Notice that savings are defined as the change in nonhuman wealth. In the context of a model with no capital gains that definition is equivalent to disposable income minus consumption – the N.I.P.A. definition. In the presence of capital gains the present definition is the relevant one and the one used in the subsequent empirical work.

If yl is $I(1)$, W will be $I(0)$ with unconditional expectation $-\mu/z(z-r) \simeq -(\mu/zp)$. Accordingly, savings will be stationary with zero mean.

Case III: $p, \alpha > 0$ (finite horizons, life-cycle savings). In this case we have $\beta < 1$, and the following expression for nonhuman wealth obtains:

$$W_t = -\frac{\Omega}{(z-r)} + \sum_{j=0}^{\infty} [(1-(z-r))]^j [(1-\beta) yl_{t-1-j} - u_{t-1-j}].$$

Thus W is a distributed lag of yl and will share the latter's low frequency features. In particular, if yl is an $I(1)$ process, W will also be $I(1)$. In that case, savings will be $I(0)$ with a well-defined unconditional expectation:

$$\mathrm{E}(s) \equiv \mathrm{E}(\Delta W) = \frac{(1-\beta)\mu}{(z-r)} \cong (1-\beta)\mu/p.$$

Notice that $(\partial \mathrm{E}(s)/\partial\alpha) > 0$ when $\mu > 0$, a property which motivates the use of the term 'life-cycle savings'. Furthermore, we can rewrite (13) as follows:

$$\Delta W_t = -(z-r) W_{t-1} + (1-\beta) yl_{t-1} - \Omega - u_{t-1}.$$

If yl is $I(1)$, we know u and ΔW are stationary, so it must be the case that W and yl are cointegrated with a cointegrating vector $[1, -(1-\beta)/(z-r)]$. In other words, the presence of life-cycle savings links the trend in labor income and that in nonhuman wealth.[10]

Table 1 summarizes the implications of finite horizons and life-cycle savings on the order of integration and the unconditional expectations of consumption, nonhuman wealth, and savings, under the maintained hypothesis of a unit root in labor income. Under that assumption aggregate savings are stationary independently of the relevant horizon and/or the presence of life-cycle savings. From this, it follows that *total* income and consumption will be cointegrated regardless of the values of p and α.

It is important to stress at this point that a common trend in yl, W, and c is not necessarily inconsistent with *any* infinite-lived representative consumer model. Indeed, a variety of representative consumer models developed in the real business-cycle literature[11] imply a balanced growth path for those variables, thus generating a similar cointegration result. In those models the

[10]It can be shown that $(\partial\Omega/\partial\alpha) < 0$ and $(\partial\beta/\partial\alpha) < 0$. Intuitively, a higher value of α implies, cetaris paribus, lower expected lifetime resources for those currently alive and thus lower consumption. When $\alpha > 0$, the W process becomes affected by the current level of yl [see eq. (13)], so that permanent changes in yl will have a permanent effect on W's 'steady-state' value and will thus generate the common trend between yl and W.

[11]See King, Plosser, and Rebelo (1988) for a survey of those models.

Table 1

Long-run properties of c, W, and s when yl has a unit root.[a]

	Assumptions on p and α[b]		
	$p=\alpha=0$	$p>0;\ \alpha=0$	$p>0;\ \alpha>0$
c	$I(1)$	$I(1)$	$I(1)$
$E(\Delta c)$	0	μ	$(\Gamma\beta/p)\mu$
$CI[c, yl]$[c]	—	$(1, -1)$	$(1, -\Gamma\beta/p)$
W	$I(1)$	$I(0)$	$I(1)$
$E(W)$	—	$-\mu/zp$	—
$CI[W, yl]$[c]	—	—	$(1, -(1-\beta)/p)$
$s\equiv\Delta W$	$I(0)$	$I(0)$	$I(0)$
$E(s)$	$-(\mu/r)$	0	$(1-\beta)\mu/p$

[a] c denotes aggregate consumption, W is aggregate nonhuman wealth, $s\equiv\Delta W$ is aggregate savings, and yl is aggregate labor income. Δ is the first-difference operator E is the unconditional expectation operator. $I(d)$ denotes integration of order d. μ is the mean of Δyl. p is the one-period-ahead death probability. r is the interest rate $z\equiv(1+r)/(1-p)$, $\Gamma\equiv[1-(1-\alpha)(1-p)]$, and $\beta\equiv z/(z+\alpha)$ α is the rate of decline in individual labor supply.

[b] Under $p>0$ finite horizons are present. Under $\alpha>0$ life-cycle savings are present.

[c] $CI[x, yl]$ denotes the cointegrating vector between x and yl

drift in consumption results from the interest rate being (on average) higher than the discount rate along the equilibrium path, i.e., they require the presence of significant intertemporal substitution effects.[12] The present model provides an alternative – though not incompatible – source for the common trends in c, yl, and W, based on life-cycle rather than intertemporal-substitution considerations.

3.3. Long-run behavior of c, yl, and W: The evidence

Fig. 1 plots the time series for aggregate labor income, consumption, and nonhuman wealth corresponding to the U.S. economy over the 1954:I–1988:III sample period. We use an updated version of the data set in Blinder and Deaton (1985). The data are quarterly, seasonally adjusted, per capita variables, measured in 1972 dollars.[13] A glance at fig. 1 suggests that

[12] Some recent work has attempted to measure the significance of intertemporal substitution in consumption. Deaton (1987) argues that relaxing the $r=\delta$ assumption and introducing intertemporal-substitution factors is not likely to reconcile the infinite-horizon model with the observed long-run features of the data, since consumption growth has remained positive even in periods characterized by persistent negative real rates. Hall (1988) and Campbell and Mankiw (1989) estimate the intertemporal elasticity of substitution in consumption to be zero or close to zero.

[13] The consumption data are for nondurables and services, excluding shoes and clothing. The original series is scaled up so that its sample mean matches the sample mean of total consumption. The nonhuman wealth data corresponds to the MPS series for household net worth. A detailed description of the way the labor income series is constructed from the NIPA data can be found in Blinder and Deaton (1985). The data set was kindly provided by Angus Deaton and Anil Kashyap.

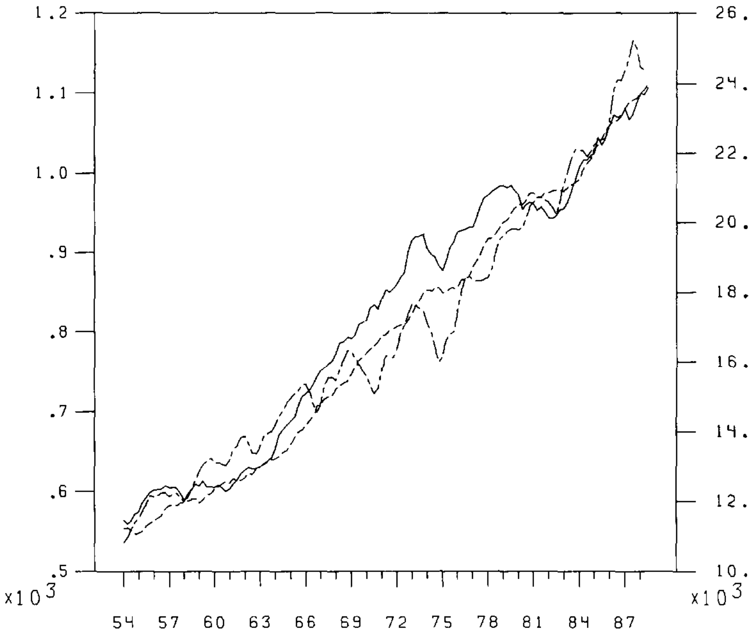

Fig. 1. Postwar U.S. time series for consumption, labor income, and nonhuman wealth. Quarterly, seasonally adjusted, per capita values, in 1972 dollars. Sample period: 1954:I–1988:III. Left scale: aggregate labor income (solid line) and aggregate consumption (short dashes). Right scale: aggregate nonhuman wealth (long dashes).

the three time series may share a (possibly stochastic) common upward trend, as is the case for many postwar U.S. series.

Tables 2, 3, and 4 formalize some of the intuition obtained by looking at fig. 1. Table 2 reports the sample means of Δc, ΔW, and Δyl, and the corresponding t-statistics. The three statistics are positive and significantly different from zero. In the life-cycle model developed above (and given $\mu > 0$), $\mathrm{E}(\Delta c) > 0$ results from finite horizons, whereas $\mathrm{E}(\Delta w) > 0$ is a consequence of life-cycle savings. Thus, in that model's context, the finding of significant positive drifts in consumption and nonhuman wealth support the idea that both finite horizons and life-cycle savings play a significant role.[14]

Table 3 reports the results of Dickey–Fuller (1979) and Phillips–Perron (1988) tests for unit roots. All the test statistics corresponding to yl, c, and W fall within the 95 percent confidence region and are thus consistent with the hypothesis of a unit root in those series. Applied to the first-differences of

[14]In the context of a similar framework, Evans (1988) and Startz (1990) develop two alternative regression-based strategies to assess the empirical relevance of finite horizons, reaching opposite conclusions. Neither paper allows for labor income decline, which may lead to misspecification of the estimated equations if $\alpha > 0$ – as the analysis in this paper suggests.

Table 2

Sample means of Δc, Δyl, and ΔW.[a]

	Sample mean	t-statistic[b]
Δc	4.05	3.67
Δyl	3.74	2.76
ΔW	96.19	3.29

[a] c denotes aggregate consumption, W is aggregate nonhuman wealth, and yl is aggregate labor income. Δ is the first-difference operator *Data:* U.S. quarterly, seasonally-adjusted, 1972 dollars, per capita variables. The sample period is 1954:I–88.III.

[b] t-statistic associated with the null of a zero mean in the variable on the left column. Computed using a four-lag Bartlett-window.

Table 3

Tests for unit roots.[a,b]

	$N(\tilde{\rho}-1)$[c]	t_ρ[d]	$Z(\tilde{\rho})$[e]	$Z(t_\rho)$[f]
yl	−4 18	−1.51	−7.24	−1.95
c	−6.11	−2.53	−8.12	−2.58
W	−5.14	−1.45	−10.24	−2.14
Δy	−95 38	−8.42	−100.59	−8.54
Δc	−103.75	−8.96	−112.22	−9.13
ΔW	−60 93	−6.13	−53.86	−5.86
Critical values[g]				
5% level	−20.7	−3.45	−20.7	−3.45
1% level	−27.4	−4.04	−27.4	−4.04

[a] c denotes aggregate consumption, W is aggregate nonhuman wealth, and yl is aggregate labor income. Δ is the first-difference operator. *Data:* U.S. quarterly, seasonally adjusted, 1972 dollars, per capita variables. The sample period is 1954:I–1988:III.

[b] All these statistics based on a OLS regression of each variable on its own lag, a time trend, and an intercept.

[c] Dickey and Fuller (1979) 'normalized bias' statistic.

[d] Dickey and Fuller (1979) t-statistic.

[e] Phillips and Perron (1988) 'normalized bias' statistic.

[f] Phillips and Perron (1988) t-statistic.

[g] Critical values correspond to the null of a unit root against a trend-stationary alternative, reported in Fuller (1976). A statistic value less than the critical value corresponds to a rejection

the same variables, the test statistics systematically reject a unit root at the 1 percent level. The evidence thus suggests that c, yl, and W are well characterized as $I(1)$ processes. Again, this is a result consistent with the presence of both finite horizons and life-cycle savings.

Table 4 reports the estimated cointegrating regressions between yl and c and between yl and W, as well as three statistics corresponding to residual-based cointegration tests. The asymptotic distribution of those statistics under the null of no cointegration is derived in Phillips and Ouliaris (1990). The formal tests do not reject a unit root in the residual of the cointegrating

Table 4

Tests for cointegration.[a]

	ξ[b]	DW.[c]	$ADF(t_\rho)$[d]	$Z(\rho)$[e]	$Z(t_\rho)$[f]
(c, yl)	1.01	0.07	−1.97	−7.44	−1.88
(yl, c)	0.96	0.07	−2.12	−8.00	−2.01
(W, yl)	21.44	0.04	−1.92	−4.39	−1.40
(yl, W)	0.04	0.04	−1.93	−4.45	−1.46
Critical values[g]					
10% level			−3.06	−17.03	−3.06
5% level			−3.36	−20.4	−3.36

[a] c denotes aggregate consumption, W is aggregate nonhuman wealth, and yl is aggregate labor income. Δ is the first-difference operator. *Data.* U.S. quarterly, seasonally adjusted, 1972 dollars, per capita variables. The sample period is 1954:I–1988:III.

[b] In (x, y) row, ξ is the OLS estimate in a cointegrating regression $x = \text{constant} + \xi y + u$.

[c] Durbin–Watson statistic for the regression described above.

[d] Augmented Dickey–Fuller test statistic applied to the fitted residuals from the cointegrating regression. See Phillips and Ouliaris (1990) for a description of the statistic.

[e] Phillips 'normalized bias' statistic applied to the fitted residuals from the cointegrating regression. See Phillips and Ouliaris (1990).

[f] Phillips t-statistic applied to the fitted residuals from the cointegrating regression. See Phillips and Ouliaris (1990).

[g] Critical values obtained from Phillips and Ouliaris (1990).

regressions, and cointegration cannot be *formally* established for either pair of variables. This result weakens the evidence for the life-cycle model, since the latter predicts cointegration. Of course, the fact that cointegration cannot be established does not imply a rejection of that model. Nevertheless, the large persistence observed in the residuals of the cointegrating regressions – reflected in Durbin–Watson statistics close to zero – suggests the existence of factors which affect the long-term behavior of the three series, but which are ignored by the model. Among candidate explanations we have: measurement error (particularly in the constructed labor income series), highly persistent fluctuations in the value of financial wealth (e.g., persistent 'bubbles'), persistent preference shocks, and liquidity constraints. Unfortunately, procedures for discriminating between those sources (or others) are not obvious, and its development is beyond the scope of this paper.

3.4. Long-run behavior and calibration of r, p, and α

The life-cycle model can be calibrated in a way consistent with the observed long-run behavior of different variables. Given (6), a natural estimate for r is given by the sample mean of $(W_{t+1} - yl_t + c_t)/W_t - 1$, which yields a value of 0.0042 (i.e., a 1.7% annual rate). Parameter p is calibrated as the reciprocal of the consumer's expected lifetime (in quarters), given a

Table 5

Calibration of parameters p, α, and β.

Expected lifetime	p[a]	$\hat{\alpha}_s$[b]	$\hat{\alpha}_w$[c]	$\hat{\beta}_s$[d]	$\hat{\beta}_w$[e]
40 years	0.0062	0.0020	0.0016	0.839	0.865
45 years	0.0055	0.0016	0.0013	0.856	0.880
50 years	0.0050	0.0014	0.0011	0.870	0.892
55 years	0.0045	0.0011	0.0009	0.882	0.902
60 years	0.0041	0.0010	0.0008	0.892	0.910

[a]One-quarter-ahead death probability consistent with the expected lifetime on the left column.

[b]Rate of decline in individual labor supply consistent with observed average savings, given 'p' on the second column

[c]Rate of decline in relative labor income consistent with the estimated cointegrating vector for W and yl, given 'p' on the second column.

[d]Marginal propensity to consume out of labor income consistent with observed average savings, given 'p' on the second column.

[e]Marginal propensity to consume out of labor income consistent with the estimated cointegrating vector for W and yl, given 'p' on the second column.

reasonable range of values for the latter. An estimate for α can then be obtained using β's definition and the previous results $\beta = 1 - (\mathrm{E}(s)p/\mu)$ or, alternatively, $\beta = 1 - p\xi_w$, where ξ_w is the coefficient in the cointegrating regression of W on yl. We can thus define the estimators $\hat{\beta}_s = 1 - (\hat{s}p/\hat{\mu})$, $\hat{\beta}_w = 1 - p\hat{\xi}_w$, $\hat{\alpha}_s \equiv z(1 - \hat{\beta}_s)/\hat{\beta}_s$, and $\hat{\alpha}_w \equiv z(1 - \hat{\beta}_w)/\hat{\beta}_w$, where $\hat{s}$, $\hat{\mu}$, and $\hat{\xi}_w$ are the available consistent estimates for $\mathrm{E}(s)$, μ, and ξ_w.

Table 5 reports the estimates of $\hat{\beta}_s$, $\hat{\beta}_w$, $\hat{\alpha}_s$, and $\hat{\alpha}_w$ obtained under alternative assumptions on the consumer's expected horizon, ranging from 40 to 60 years. The estimates have a reasonable order of magnitude and do not seem too sensitive to the horizon nor the procedure chosen. Estimates of α – the rate of decline in relative labor income – range between 0.0008 and 0.002 (0.3% and 0.8% at annual rates). Estimates of β – the marginal propensity to consume out of aggregate labor income in (9) – range between 0.83 and 0.91.

4. Finite horizons, life-cycle savings, and consumption puzzles

4.1. Predictability of consumption in the life-cycle model

In contrast with the representative consumer model [Hall (1978)], the life-cycle model implies that changes in aggregate consumption should be predictable. This can be easily seen by rewriting eq. (10) as follows:

$$\mathrm{E}_{t-1}\Delta c_t = -pc_{t-1} + \Gamma\beta yl_{t-1} + \Gamma(1+z)(1-\alpha)^{-1}[\Omega + u_{t-1}]. \tag{10'}$$

If $p = \alpha = 0$, then $\Gamma = 0$ and eq. (10′) collapses to the martingale result. However, to the extent that $p > 0$ (and therefore $\Gamma > 0$), changes in aggregate consumption will be predictable by lagged labor income, lagged consumption, and any lagged variable correlated with u_{t-1} (i.e., with lagged expectations of changes in future aggregate labor income).[15] However, the predictability of consumption implied by the life-cycle model is not unrestricted: (10′) actually imposes restrictions on that predictability. Given a univariate process for aggregate labor income, such restrictions can be tested econometrically. To illustrate this, assume that aggregate labor income changes follow the AR(1) process $(1 - \rho L)\,\Delta yl_t = \mu(1-\rho) + \varepsilon_t$. This simple model captures pretty well the serial correlation properties of labor-income changes and has been used by many authors [e.g. Deaton (1987)]. The life-cycle model implies a regression equation of the form:

$$\Delta c_t = a_0 + a_1 c_{t-1} + a_2 yl_{t-1} + a_3 \Delta yl_{t-1} + X_{t-1}\lambda + \eta_t, \qquad (14)$$

where $\mathrm{E}_{t-1}\eta_t = 0$, X_{t-1} is a $(1 \times m)$ vector containing any stationary variable known as of period $t-1$ (other than Δyl_{t-1} or a stationary combination of c_{t-1} and yl_{t-1}), and λ is a $(m \times 1)$ parameter vector. Under the AR(1) assumption for Δyl the model predicts that X_{t-1} should not appear in (14), i.e., λ should be zero.

That restriction can be easily tested econometrically. Table 6 reports the results of such a test. We specify $X \equiv (\Delta yl_{t-2}, \Delta yl_{t-3}, \Delta yl_{t-4})$. Row (1) in that table reports the estimates of the regression of Δc on four lags of Δyl. Under the infinite-horizon model ($p = \alpha = 0$) all the estimated coefficients should be jointly insignificant, but the F statistic clearly rejects that hypothesis at a very low significance level. This rejection of the infinite-horizon model corresponds, essentially, to the findings in Flavin (1981) and others.

Row (2) shows the coefficient estimates of a similar regression, but now lagged c and lagged yl are also included as regressors, thus allowing for finite-horizon effects. As discussed above, under the AR(1) assumption for Δyl, the life-cycle model predicts that the coefficients on Δyl_{t-2}, Δyl_{t-3}, and Δyl_{t-4} should be zero. Interestingly, even though yl and c are $I(1)$ series, the OLS estimates of those coefficients are asymptotically normal under the life-cycle model[16] [West (1988), Sims, Stock, and Watson (1990)], which allows us to test the $\lambda = (0,0,0)'$ null with the usual F statistic. The value of

[15]Notice that the predictability of consumption changes does not hinge on the presence of declining labor supply over the individual consumer's lifetime ($\alpha > 0$) but on *finite horizons*.

[16]Note that (14) can be rewritten as $\Delta c_t = \text{constant} + a_1[c_{t-1} - \xi_c\, yl_{t-1}] + (a_1\xi_c + a_2)yl_{t-1} + a_3(\Delta yl_{t-1} - \mu) + \sum_{i>1}\lambda_i(\Delta yl_{t-i} - \mu) + \eta_t$, where $(1, -\xi_c)$ is the cointegrating vector between c and yl implied by the model (see table 1) and $\mu \equiv \mathrm{E}(\Delta yl) > 0$ as shown above (table 2). Such a 'transformed' regression corresponds to the regression model analyzed by West (1988) and for which OLS estimates are asymptotically normal.

Table 6

Predictability of consumption in the life-cycle model.[a]

Dependent variable: Δc_t

Explanatory variables:

	Constant	c_{t-1}	yl_{t-1}	Δyl_{t-1}	Δyl_{t-2}	Δyl_{t-3}	Δyl_{t-4}	F
(1)[b]	2.736	—	—	0.095	−0.026	0.096	0.216	15.72[d]
	(1.461)			(0.011)	(0.011)	(0.041)	(0.044)	
(2)[c]	−0.854	−0.009	0.013	0.087	−0.037	0.085	0.201	12.93[d]
	(1.348)	(0.010)	(0.010)	(0.038)	(0.039)	(0.040)	(0.039)	

[a] c is aggregate consumption and yl is aggregate labor income. Δ is the first-difference operator. *Data:* U.S. quarterly, seasonally adjusted, 1972 dollars, per capita variables. The sample period is 1954:I–1988:III.

[b] OLS regression of Δc on four lags of Δyl. Standard errors in brackets. F statistic corresponds to the null that the coefficients on the four lags of Δyl are all zero. Sample period is 1955:I–1988:III.

[c] OLS regression of Δc on lagged c, lagged yl, and four lags of Δyl. Standard errors in brackets. F statistic corresponds to the null that the coefficients on Δyl_{t-2}, Δyl_{t-3}, and Δyl_{t-4} are all zero. The sample period is 1955:I–1988:III.

[d] Significant at less than 1 percent level.

F obtained is highly significant, thus rejecting the simple restriction on the predictability of consumption implied by the life-cycle model. In other words, even though the life-cycle model implies that consumption changes should be predictable, the evidence suggests that much of that observed predictability cannot be accounted for life-cycle factors, i.e., consumption changes appear to be 'too predictable'.

4.2. *Variability of consumption in the life-cycle model*

Recent evidence using postwar U.S. data suggests that consumption is smoother than is implied by the infinite-lived representative consumer model [Deaton (1987), West (1988a), Campbell and Deaton (1989)].[17] The life-cycle model developed above suggests that the finding of excess smoothness in consumption may be spurious, since it may arise from neglecting the role of finite horizons and life-cycle savings. Eq. (9) above implies

$$\eta_t = \beta \sum_{j=0}^{\infty} (1+z)^{-j}(1-\alpha)^j(\mathrm{E}_t - \mathrm{E}_{t-1})\Delta yl_{t+j}, \tag{16}$$

[17] Attempts to explain the excess-smoothness puzzle by relaxing some of the assumptions in the standard infinite-horizon model can be found in Christiano (1987), Quah (1990), Diebold and Rudebusch (1990), Caballero (1990), Gali (1990), and Zeldes (1989), among others.

where η_t is the innovation in aggregate consumption resulting from news about future labor income.

Assuming Δyl follows the univariate AR(1) process introduced above we can rewrite (16) as follows [Hansen and Sargent (1981)]:

$$\eta_t = \Psi \varepsilon_t, \tag{17}$$

where $\Psi \equiv \beta(1+z)/[(1+z)-(1-\alpha)\rho]$. If $p, \alpha > 0$ (and, accordingly, $z > r$ and $\beta < 1$), the innovation in consumption resulting from an innovation in labor income predicted by the life-cycle model is smaller than that predicted by the infinite-horizon model ($p = \alpha = 0$, $z = r$, $\beta = 1$). This results from the wedge that finite horizons and life-cycle savings introduce between (a) the revisions in expectations of future aggregate labor income and (b) the revisions in expected labor income accruing in the future to those who are currently alive, *i.e., those whose decisions affect current aggregate consumption.*

Using $\sigma(x)$ denotes the standard deviation of x, we have that $\sigma(\Delta c) \geq \sigma(\eta)$ by construction. Thus, (17) implies

$$\Phi \equiv \sigma(\Delta c)/\sigma(\varepsilon) \geq \Psi. \tag{18}$$

The previous inequality can be tested econometrically, given the values for p, r, α, and z obtained above.[18] Table 7 reports the results of the test. Φ, the ratio of standard deviations on the left-hand side of (18), is estimated to be 0.54, less than half the estimated value for Ψ, the lower bound implied by the life-cycle model. Estimates of the latter range from 1.23 to 1.31. As indicated by the t-statistic the difference is highly significant. Thus, the evidence suggests that consumption is much smoother than a model allowing for finite horizons and life-cycle savings predicts. Even though both factors decrease the variability of consumption consistent with optimal behavior by consumers, reasonable parameterizations of the model consistent with the long-run properties of the data still imply a standard deviation for aggregate consumption more than twice the size of the observed standard deviation.[19]

[18]A similar test is carried out in West (1988a) in the context of a representative consumer model. The test uses a GMM procedure [Hansen (1982)] with the just-identifying orthogonality conditions given by

$$\mathrm{E}\begin{bmatrix} [(\Delta yl_t - \mu) - \rho(\Delta yl_{t-1} - \mu)]\,\Delta yl_{t-1} \\ [(\Delta yl_t - \mu) - \rho(\Delta yl_{t-1} - \mu)]^2 - \sigma^2(\varepsilon) \\ (\Delta c_t - \mathrm{E}(\Delta c))^2 - \sigma^2(\Delta c) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \text{all } t.$$

For computational simplicity, I assume that $\mathrm{E}(\Delta c)$ and μ are known and equal to their sample counterparts. Thus the parameter vector estimated is $[\rho, \sigma^2(\varepsilon), \sigma^2(\Delta c)]'$. A test for 'excess smoothness' can then be constructed as a one-sided test of the nonlinear restriction implied by (18) on the elements of that vector, with the inequality sign replaced by an equality sign.

[19]Using a traditional life-cycle model, Clarida (1988) reaches a similar conclusion when he allows for serial correlation in Δyl

Table 7

Excess smoothness in the life-cycle model.[a]

Estimated AR(1) model for Δyl: $\Delta yl_t = \underset{(0.65)}{2.65} + \underset{(0.08)}{0.323}\Delta yl_{t-1} + \varepsilon_t$, $D.W. = 2.01$

Additional estimates:[b] $\hat{\sigma}(\varepsilon) = 6.89$, $\hat{\sigma}(\Delta c) = 3.73$, $\hat{\Phi} \equiv \hat{\sigma}(\Delta c)/\hat{\sigma}(\varepsilon) = 0.54$, $\hat{r} = 0.0044$

Excess smoothness tests: Expected lifetime	Ψ[c]	$(\Psi - \hat{\Phi})$	t-statistic[d]
40 years	1.23	0.69	7 42
50 years	1.28	0.74	7.62
60 years	1.31	0.77	7.75

[a] c is aggregate consumption and yl is aggregate labor income. Δ is the first-difference operator. Data: U.S. quarterly, seasonally adjusted, 1972 dollars, per capita variables The sample period is 1954:I–1988:III

[b] $\hat{\sigma}(x)$ denotes variable x's sample standard deviation.

[c] Ψ denotes the variance ratio $\sigma(\Delta c)/\sigma(\varepsilon)$ predicted by the life-cycle model under the estimated AR(1) model for Δyl, $r = 0.0044$, and the β_s value reported in table 5 for each assumption on expected lifetime.

[d] The t-statistic corresponds to the null $\hat{\Phi} = \Psi$.

5. Summary and conclusions

In this paper we have developed an explicitly aggregated life-cycle model which preserves much of the tractability and easy econometric implementation that characterize the representative consumer model.

The life-cycle model generates a common upward trend in aggregate consumption, labor income, and financial wealth. That trend is consistent with individual consumption being a martingale (without drift). The model also implies that aggregate consumption changes should be predictable and smoother than the representative consumer model predicts. Thus, the life-cycle factors appear as a potential explanation of some of the puzzles examined in the recent consumption literature. However, formal empirical tests applied to postwar U.S. data suggest that neither the predictability nor the smoothness of consumption changes can be fully accounted for by life-cycle considerations. Further research is needed to pin down the source(s) of departures from the model's predictions.

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